

ARE ALL HADRONS ALIKE? ELECTROPRODUCTION of RESONANCES at LARGE MOMENTUM TRANSFERS

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- T.-S.H. Lee *et al.* Research Proposal: Nucleon Resonance Studies with CLAS12

- ① why we need large Q^2 ?
- ② which resonances to study?
- ③ physics goals?
- ④ theory perspectives?

- In this talk I argue:

- ① \Rightarrow to select well-defined parton (valence) configurations
- ② \Rightarrow concentrate on parity partners
- ③ \Rightarrow momentum fraction distributions of valence quarks and effects of orbital angular momentum
- ④ \Rightarrow combination of lattice calculations and light-cone sum rules



**Spontaneous chiral symmetry breaking lifts
degeneracy between**

$$N(940), J^P = \frac{1}{2}^+ \quad N^*(1535), J^P = \frac{1}{2}^-$$

How is this reflected in their parton distributions?

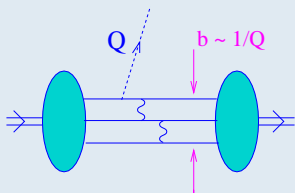


QCD factorization

In theory

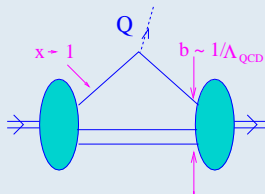
Efremov, Radyushkin, Brodsky, Lepage, Chernyak

- quarks can acquire large transverse momenta when they exchange gluons
- “hard” gluon exchanges can be separated from “soft” nonperturbative wave functions
- hard gluons can only be exchanged at small transverse separations



Hard rescattering:

Small b
Average $0 < x < 1$



Soft (Feynman):

Average b
Large $x \rightarrow 1$

In practice three-quark states indeed seem to dominate, however

- “Squeezing” to small transverse separations occurs very slowly
- Helicity selections rules do not work. Orbital angular momentum?
- \Rightarrow More complicated nonperturbative input needed



Wave functions and Distribution amplitudes

• Nucleon light-cone wave function

Brodsky, Lepage

$$|P \uparrow\rangle^{\ell_z=0} = \int \frac{[dx][d^2\vec{k}]}{12\sqrt{x_1 x_2 x_3}} \psi^{L=0}(x_i, \vec{k}_i) \times \left\{ \left| u^\uparrow(x_1, \vec{k}_1) u^\downarrow(x_2, \vec{k}_2) d^\uparrow(x_3, \vec{k}_3) \right\rangle - \left| u^\uparrow(x_1, \vec{k}_1) d^\downarrow(x_2, \vec{k}_2) u^\uparrow(x_3, \vec{k}_3) \right\rangle \right\}$$

• Leading-twist-three distribution amplitude

Brodsky, Lepage, Peskin, Chernyak, Zhitnitsky

$$\Phi_3(x_1, x_2, x_3; \mu) = 2 \int [d^2\vec{k}] \psi^{L=0}(x_1, x_2, x_3; \vec{k}_1, \vec{k}_2, \vec{k}_3)$$

can be studied using the OPE

$$\begin{aligned} \Phi_3(x_i; \mu) = & 120 f_N x_1 x_2 x_3 \left\{ 1 + c_{10} (x_1 - 2x_2 + x_3) L^{\frac{8}{3\beta_0}} \right. \\ & + c_{11} (x_1 - x_3) L^{\frac{20}{9\beta_0}} + c_{20} \left[1 + 7(x_2 - 2x_1 x_3 - 2x_2^2) \right] L^{\frac{14}{3\beta_0}} \\ & \left. + c_{21} (1 - 4x_2) (x_1 - x_3) L^{\frac{40}{9\beta_0}} + c_{22} \left[3 - 9x_2 + 8x_2^2 - 12x_1 x_3 \right] L^{\frac{32}{9\beta_0}} + \dots \right\} \end{aligned}$$

• $f_N(\mu_0)$: wave function at the origin

• $c_{nk}(\mu_0)$: shape parameters

$$L \equiv \alpha_s(\mu) / \alpha_s(\mu_0)$$



Braun, Manashov, Rohwild

Wave functions and Distribution amplitudes (II)

- Contributions of orbital angular momentum

Ji, Ma, Yuan, '03

$$|P \uparrow\rangle^{\ell_z=1} = \int \frac{[dx][d^2\vec{k}]}{12\sqrt{x_1 x_2 x_3}} \left[k_1^+ \psi_1^{L=1}(x_i, \vec{k}_i) + k_2^+ \psi_2^{L=1}(x_i, \vec{k}_i) \right] \times \\ \times \left\{ \left| u^\uparrow(x_1, \vec{k}_1) u^\downarrow(x_2, \vec{k}_2) d^\downarrow(x_3, \vec{k}_3) \right\rangle - \left| d^\uparrow(x_1, \vec{k}_1) u^\downarrow(x_2, \vec{k}_2) u^\downarrow(x_3, \vec{k}_3) \right\rangle \right\}$$

are related to higher-twist-four distribution amplitudes

Belitsky, Ji, Yuan, '03

$$\Phi_4(x_2, x_1, x_3; \mu) = 2 \int^\mu \frac{[d^2\vec{k}]}{m_N x_3} k_3^- \left[k_1^+ \psi_1^{L=1} + k_2^+ \psi_2^{L=1} \right](x_i, \vec{k}_i)$$

$$k^\pm = k_x \pm ik_y$$

$$\Psi_4(x_1, x_2, x_3; \mu) = 2 \int^\mu \frac{[d^2\vec{k}]}{m_N x_2} k_2^- \left[k_1^+ \psi_1^{L=1} + k_2^+ \psi_2^{L=1} \right](x_i, \vec{k}_i)$$

and, again, can be studied using OPE

Braun, Fries, Mahnke, Stein '00

$$\Phi_4(x_i; \mu) = 12\lambda_1 x_1 x_2 + 12f_N x_1 x_2 \left[1 + \frac{2}{3}(1 - 5x_3) \right] + \dots$$

$$\Psi_4(x_i; \mu) = 12\lambda_1 x_1 x_3 + 12f_N x_1 x_3 \left[1 + \frac{2}{3}(1 - 5x_2) \right] + \dots$$

- to this accuracy only one new nonperturbative constant $\lambda_1(\mu)$



What is to be done?

Braun *et al.* Phys.Rev.Lett.103:072001,2009

- Calculate moments of distribution amplitudes (DAs)
⇐ **lattice QCD**

- expensive
- many technical problems still need to be solved
- only limited information
- studies of parity partners look most promising, e.g.

$$\langle 0|qqq|N(p)\rangle = f_N N(p) \quad \langle 0|qqq|N^*(p)\rangle = f_{N^*} \gamma_5 N(p)$$

- Calculate electroproduction cross sections (transition form factors) in terms of DAs
⇐ **light-cone sum rules (LCSRs)**

- based on analyticity and quark-hadron duality
- well-known and tested technique for mesons, less so for baryons
- irreducible uncertainty of 20%(?) – need confirmation
- NLO calculations so far not available



Parity separation

$$32^3 \times 64$$

$$a = 0.0753 \text{ fm}$$

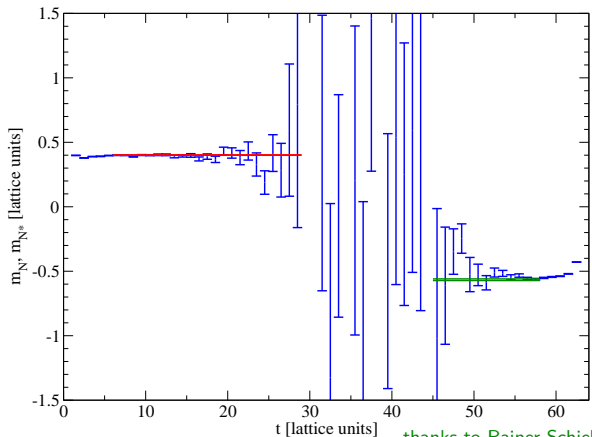
$$a^{-1} = 2.620 \text{ GeV}$$

$$m_\pi = 282(2) \text{ MeV}$$

$$m_\pi L = 3.44$$

$$m_N = 1051(5) \text{ MeV}$$

$$m_{N^*} = 1482(17) \text{ MeV}$$



- **New:** Generalized Lee-Leinweber parity projectors implemented



Overview of lattices

$N_f = 2$ clover Wilson fermions

κ	m_π / MeV	Size	# Configurations
$\beta = 5.29, a = 0.0753 \text{ fm}$			
0.13590	627	$24^3 \times 48$	901
0.13620	407	$24^3 \times 48$	850
0.13632	282	$32^3 \times 64$	578 + more in progress
0.13632	271	$40^3 \times 64$	in progress
0.13640	170	$40^3 \times 64$	coming soon
$\beta = 5.40, a = 0.0672 \text{ fm}$			
0.13610	648	$24^3 \times 48$	687
0.13625	558	$24^3 \times 48$	1180
0.13640	451	$24^3 \times 48$	1037
0.13660	233	$48^3 \times 64$	coming soon

[QCDSF collaboration]



Wave functions at the origin

$$32^3 \times 64$$

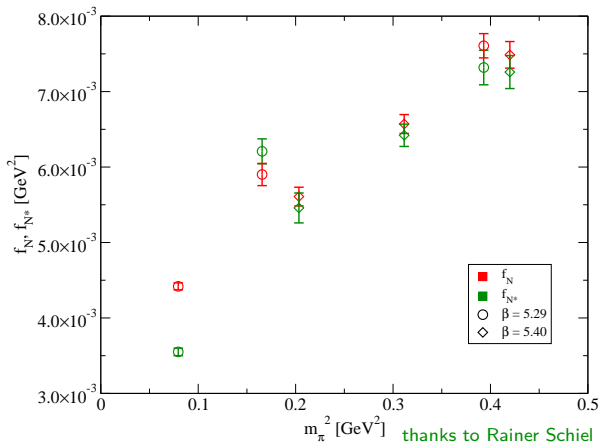
$$a = 0.0753 \text{ fm}$$

$$m_\pi = 282(2) \text{ MeV}$$

$$m_\pi L = 3.44$$

$$f_N = 4.42(5) \cdot 10^{-3} \text{ GeV}^2$$

$$f_{N^*} = 3.55(5) \cdot 10^{-3} \text{ GeV}^2$$



- All results preliminary, statistical errors only, no chiral extrapolation



Shape parameters

$$32^3 \times 64$$

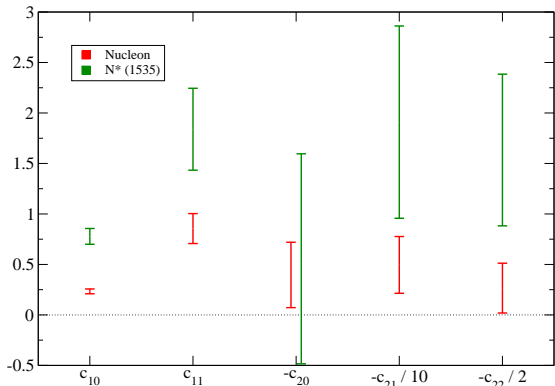
$$a = 0.0753 \text{ fm}$$

$$m_\pi = 282(2) \text{ MeV}$$

$$m_\pi L = 3.44$$

$$578 \times 4$$

configurations

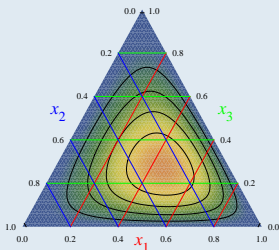


thanks to Rainer Schiel

- All results preliminary, statistical errors only, no chiral extrapolation



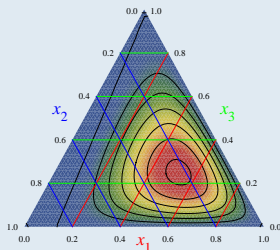
Shape parameters (II)



NUCLEON

$$c_{10}^N = 0.233 \pm 0.024,$$

$$c_{11}^N = 0.855 \pm 0.145,$$



$N^*(1535)$

$$c_{10}^{N^*} = 0.778 \pm 0.078$$

$$c_{11}^{N^*} = 1.84 \pm 0.41$$

thanks to Rainer Schiel

- Momentum fraction distribution of valence quarks in $N^*(1535)$ is much more asymmetric as compared to the nucleon
- All results preliminary, statistical errors only, no chiral extrapolation



Open problems

- **Energy conservation on a lattice**

$$\partial(A \cdot B) = (\partial A) \cdot B + A \cdot (\partial B) + \mathcal{O}(a)$$

In our latest data

$$x_1 + x_2 + x_3 \simeq 0.94$$

Becomes a serious issue for second moments

- **Calculations of third, fourth etc. moments not feasible**
(nonperturbative operator renormalization needed)
- **Decay width $\Gamma \sim 150 \text{ MeV}$, $N^*(1535) \rightarrow N\pi, N\eta$**
- **Contamination by $N^*(1650)$**



From distribution amplitudes to form factors: Duality

$$R(s = E_{\text{cm}}^2) = \frac{\begin{array}{c} e^+ \\ \swarrow \\ \text{---} \\ \searrow \\ e^- \end{array} \rightarrow \begin{array}{c} q \\ \swarrow \\ \text{---} \\ \searrow \\ \bar{q} \end{array} \rightarrow \text{hadrons}}{\begin{array}{c} e^+ \\ \swarrow \\ \text{---} \\ \searrow \\ e^- \end{array} \rightarrow \begin{array}{c} \mu^+ \\ \swarrow \\ \text{---} \\ \searrow \\ \mu^- \end{array}}$$

observe

$$R^{\text{QCD}}(s) \neq R^{\text{exp}}(s)$$

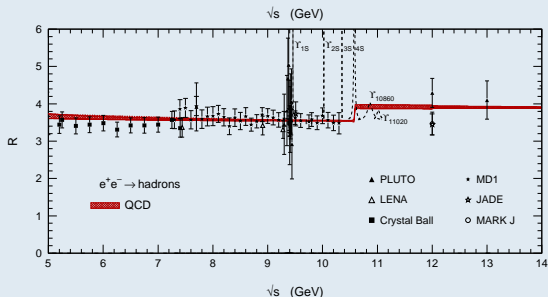
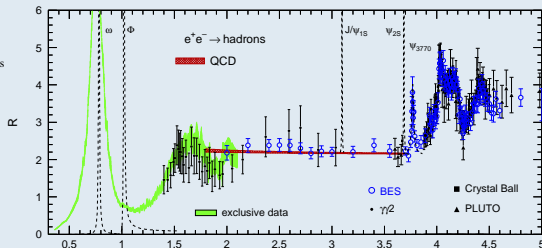
in the resonance region

$\sqrt{s} < 1.5 \text{ GeV}$, but

$$\int_0^{s_0} ds R^{\text{QCD}}(s) = \int_0^{s_0} ds R^{\text{exp}}(s)$$

s_0 is called

interval of duality



Davier et al., *Eur.Phys.J.C*27:497-521,2003



a consequence of two major principles:

- unitarity ← probability interpretation of wave functions

$$R(s) = \frac{1}{\pi} \text{Im} \Pi(s = q^2)$$

where

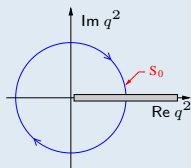
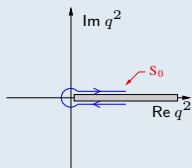
$$i \int d^4x e^{iqx} \langle 0 | T \{ j_\mu^{\text{em}}(x) j_\nu^{\text{em}}(0) \} | 0 \rangle = (q_\mu q_\nu - g_{\mu\nu} q^2) \Pi(q^2)$$

- analyticity ← causality

$$R(s) = \frac{1}{2\pi i} [\Pi(q^2 + i\epsilon) - \Pi(q^2 - i\epsilon)]$$

$$\int_0^{s_0} ds R(s) = \frac{1}{2\pi i} \oint dq^2 R(q^2)$$

$$\simeq \frac{1}{2i} \oint dq^2 R^{\text{pQCD}}(q^2)$$



because the region of $q^2 \sim \Lambda_{\text{QCD}}^2$ is avoided



can be used to estimate hadron properties

example: pion decay constant

$$\langle 0 | A_\mu(0) | \pi^+(p) \rangle = i p_\mu f_\pi, \quad A_\mu = \bar{u} \gamma_\mu \gamma_5 d$$

consider

$$i \int d^4x e^{iqx} \langle 0 | T \{ A_\mu(x) A_\nu^\dagger(0) \} | 0 \rangle = (q_\mu q_\nu - g_{\mu\nu} q^2) \Pi_A(q^2)$$

- physical spectral density: $R_A^{\text{phys}}(s) = f_\pi^2 \delta(s - m_\pi^2) + \text{resonances} + \text{continuum}$
- QCD spectral density: $R_A^{\text{QCD}}(s) = \frac{1}{4\pi^2} + \text{corrections}$

equating $\int_0^{s_0} ds R_A^{\text{phys}}(s) = \int_0^{s_0} ds R_A^{\text{QCD}}(s)$ obtain a duality relation:

$$s_0 = 4\pi^2 f_\pi^2, \quad s_0 \sim 0.7 \text{ GeV}^2$$

IMPORTANT: $s_0 \gg \Lambda_{\text{QCD}}^2 \sim (0.2 \text{ GeV})^2; \quad \alpha_s(s_0)/\pi \ll 1$

- refinement \Rightarrow QCD sum rule approach

*Shifman, Vainstein, Zakharov, Nucl.Phys.B147:385-447,1979
3646 citations in SLAC SPIRES, as of 13.05.10*



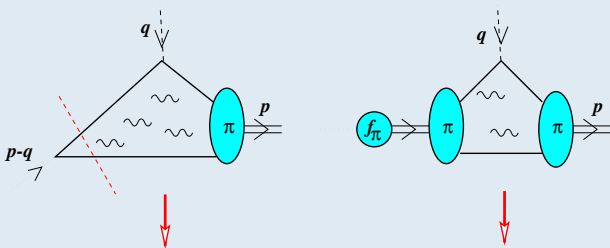
Light-Cone Sum Rules

Example: pion form factor

- another refinement \Rightarrow **Light-Cone Sum Rules**:

$$T_{\mu\nu}(p, q) = i \int dx e^{-iqx} \langle 0 | T \{ A_\nu(0) j_\mu^{\text{em}}(x) \} | \pi(p) \rangle = 2p_\mu p_\nu T(p, q) + \dots$$

duality:



$$\frac{1}{\pi} \int_0^{s_0} ds \operatorname{Im} T(p, q)$$

duality
=

$$f_\pi F_\pi(Q^2)$$

- $T(p, q)$ is calculated in terms of distribution amplitudes of increasing twist

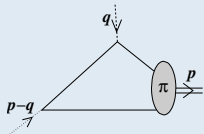
Balitsky, Braun, Kolesnichenko, Nucl.Phys.B312:509-550,1989

Braun, Halperin, Phys.Lett.B328:457-465,1994



- Leading-order calculation

$$T(p, q) = \int_0^1 dx \frac{x f_\pi \phi_\pi(x)}{(1-x)Q^2 - xs - i\epsilon}, \quad s = (p - q)^2$$



$$\frac{1}{\pi} \text{Im } T(s) = f_\pi \int_0^1 dx x \phi_\pi(x) \delta((1-x)Q^2 - xs) = \frac{f_\pi Q^2}{(Q^2 + s)^2} \phi_\pi\left(x = Q^2 / (Q^2 + s)\right)$$

leading to

$$F_\pi(Q^2) = \frac{1}{\pi f_\pi} \int_0^{s_0} ds \text{Im } T(s) = \int_0^1 dx \phi_\pi(x) \frac{Q^2}{Q^2 + s_0}$$

- integral over the end-point region \hookrightarrow purely soft contribution
- $\phi_\pi(x) \sim (1-x)$ for $x \rightarrow 1 \hookrightarrow$ suppression
- for asymptotic DA $\phi_\pi(x) = 6x(1-x)$ (example)

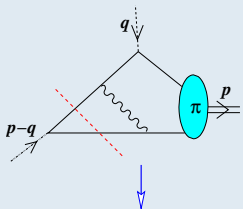
$$F_\pi^{\text{soft}}(Q^2) = \frac{s_0^2}{(Q^2 + s_0)^2} \left[1 + 2Q^2 / (Q^2 + s_0) \right], \quad F_\pi^{\text{hard}}(Q^2) = \frac{8\pi f_\pi^2 \alpha_s(Q^2)}{Q^2} = \frac{\alpha_s}{\pi} \frac{2s_0}{Q^2}$$

- loose s_0/Q^2 but win α_s/π
- note $s_0/Q^2 \gg \Lambda_{\text{QCD}}^2/Q^2$ (factor 10)

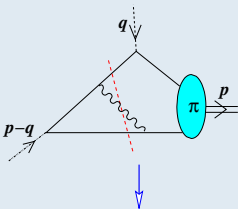


different dispersion parts:

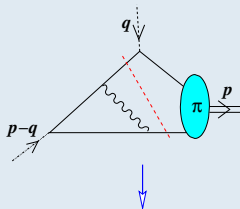
radiative corrections



hard rescattering



qGq component in the WF



initial state interaction

♥ LCSRs are fully consistent with pQCD

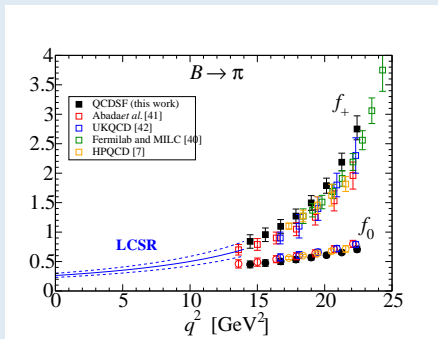
- a complicated interplay of soft and hard contributions;

*explicit calculation suggests significant cancellations
between soft and hard higher-twist corrections*

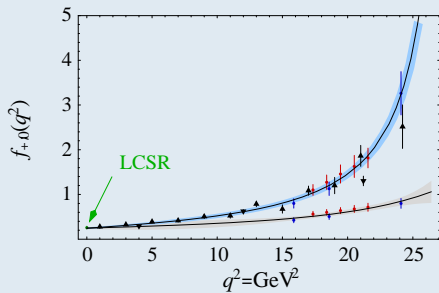
Braun, Khodjamirian, Maul, Phys.Rev.D61:073004,2000



Light-Cone Sum Rules: $B \rightarrow \pi \nu_\ell$, state-of-the-art



Ball, Zwicky, *Phys.Rev.D*71:014015,2005
 Duplancic, *et al.*, *JHEP* 0804:014,2008



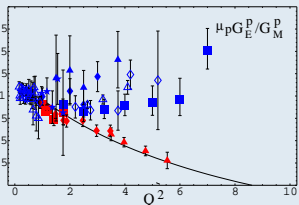
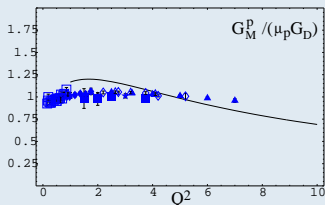
Flynn, Nieves, *Phys.Rev.D*76:031302,2007

$$|V_{ub}| = (3.5 \pm 0.4 \pm 0.2 \pm 0.1) \times 10^{-3}$$

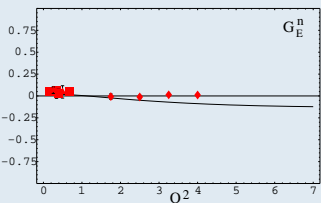
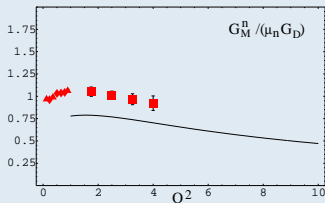


Light-Cone Sum Rules: Nucleon Electromagnetic Formfactors

proton



neutron



Braun, Lenz, Wittmann; PRD73:094019,2006

- Nucleon DAs fitted to the G_E^p / G_M^p ratio



Electroproduction $\gamma^* N \rightarrow N^*(1535)$

electromagnetic transition matrix element

$$\langle N^*(P') | j_\nu^{\text{em}} | N(P) \rangle = \bar{N}^*(P') \left(\frac{G_1(q^2)}{m_N^2} (\not{q} q_\nu - q^2 \gamma_\nu) - i \frac{G_2(q^2)}{m_N} \sigma_{\nu\rho} q^\rho \right) \gamma_5 N(P)$$

Aznauryan, Burkert, Lee, arXiv:0810.0997

helicity amplitudes

$$A_{1/2}(Q^2) = -e B \left[Q^2 G_1(Q^2) - m_N (m_{N^*} - m_N) G_2(Q^2) \right]$$

$$S_{1/2}(Q^2) = \frac{e}{\sqrt{2}} B C \left[(m_{N^*} - m_N) G_1(Q^2) + m_N G_2(Q^2) \right]$$

with

$$B = \sqrt{\frac{Q^2 + (m_{N^*} + m_N)^2}{2m_N^5(m_{N^*}^2 - m_N^2)}} \quad C = \sqrt{1 + \frac{(Q^2 - m_{N^*}^2 + m_N^2)^2}{4Q^2 m_{N^*}^2}}$$

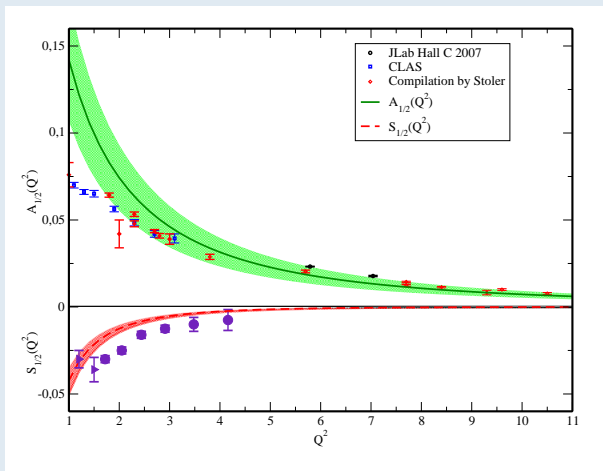


$\gamma^* N \rightarrow N^*(1535)$: helicity amplitudes

- A pilot project:**

Braun *et al.* Phys.Rev.Lett.103:072001,2009

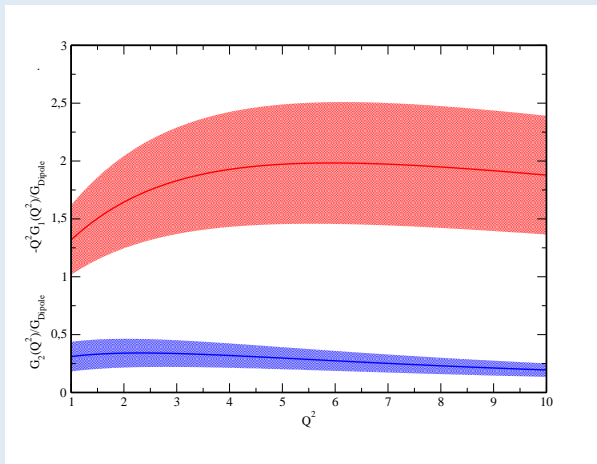
Electroproduction of $N^*(1535)$ with lattice-constrained N^* distribution amplitudes



CLAS $S_{1/2}$ data: I.G. Aznauryan *et al.*, Phys.Rev.C80:055203,2009



$\gamma^* N \rightarrow N^*(1535)$: helicity amplitudes



- Both form factors show a dipole behavior
- Negative $S_{1/2}$ implies smallness of $G_2(Q^2)$



Open problems

- **Next-to-leading-order (NLO) corrections needed**

general structure of the $Q^2 \rightarrow \infty$ expansion

$$1 \cdot \frac{1}{Q^6} + \underbrace{\frac{\alpha_s(s_0)}{\pi} \cdot \frac{1}{Q^6}}_{\uparrow \text{ LCSR}} + \left(\frac{\alpha_s(Q^2)}{\pi} \right)^2 \cdot \frac{1}{Q^4} \quad \uparrow \text{ pQCD}$$

this is not a straightforward calculation

- **Kinematic power corrections proportional to powers of m_{N^*}**

$$1 + \frac{m_{N^*}^2}{s_0} + \dots, \quad s_0 \sim 2.5 \text{ GeV}^2$$

resummation ?



First baryon LCSRs with NLO corrections

Pasek-Kumericki, Peters, Phys.Rev.D78:033009,2008

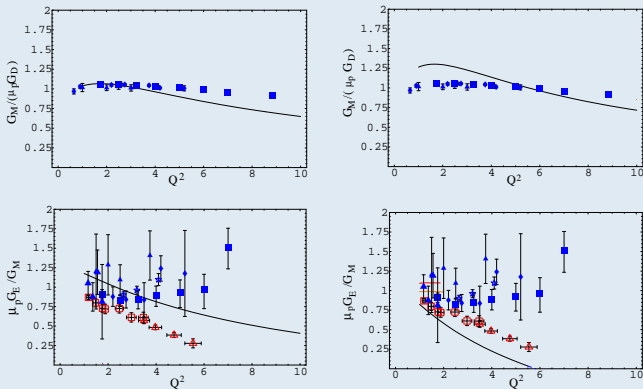


Figure: LCSR results for the electromagnetic proton form factors for a realistic model of nucleon distribution amplitudes. Left panel: Leading order (LO); right panel: next-to-leading order (NLO) for twist-three contributions. Figure adapted from [PasekKumericki:2008sj].



- Combination of lattice QCD and light-cone sum rules offers one a powerful method to study transition region to perturbative QCD

Goal

Valence quark distributions in nucleon resonances

- **Pilot project:** $\gamma^* N \rightarrow N^*(1535)$
- **Lattice:**
 - new lattices, $m_\pi \sim 280 MeV$
 - new code
 - improved parity separation implemented
- **LCSR:**
 - first results on NLO corrections
 - new DFG project 9209475



Disclaimer

From Wikipedia:

Pilot project is a short and/or incomplete realization of a certain method or idea(s) to demonstrate its feasibility, or a demonstration in principle, whose purpose is to verify that
some concept or theory is probably capable of being useful

We have a long way to go!

